Turbulence Calibration of a Hot Wire Anemometer

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Introduction

THE standard technique[1,2] for calibrating a hot wire is to plot the anemometer bridge voltage response as a function of the mean flow velocity U as measured with a pitot tube and micromanometer. This mean flow response is assumed to obey some variation of the King's Law response curve for flow over a cylinder and the graph of E vs U is differentiated to obtain dE/dU. It is assumed that the anemometer response to dynamic turbulent fluctuations is sufficiently represented by the static mean flow calibration for the purpose of calculating the turbulent characteristics of the flow.

The criticism of this approach is that a law of heat transfer based on steady flow observations is assumed to correctly relate voltage fluctuations to velocity fluctuations in the unsteady turbulent flow. This criticism was made over 20 years ago by Vernotte.[3] The recent research reported in Ref. 4 has shown that $\Delta E/\Delta u$ as measured with an oscillating wire does not agree with the value of dE/dU obtained by differentiating the King's Law response curve.

In the research reported herein, the wire is exposed to an unsteady but deterministic flow pattern and the heat-transfer characteristics are determined under these conditions. The response of the anemometer bridge voltage to the known velocity perturbation is expanded in a Taylor series rather than assuming a King's Law type response. Detection of the wire signal with a lock-in amplifier gives a measurement of the terms in the Taylor series response of bridge voltage as a function of velocity perturbation up to second order.

In the previous research,[4] the response of the anemometer to the known velocity perturbation and any random background turbulence was all lumped into an approximate term $\Delta E/\Delta u$. In the research reported herein, the linear response $\partial E/\partial u$, the first term of the nonlinear response $\partial^2 E/\partial u^2$, and the response to the level of random backround turbulence are all separated through the technique of lock-in detection.[5,6]

Theory

Figure 1 shows a schematic of a hot wire probe mounted in a turbulent flow of constant mean velocity U and exposed to a simple harmonic motion velocity perturbation $u = \omega R \cos \omega t$. The constant temperature anemometer bridge voltage response to this perturbation $E(U+u)$ may be expanded in a Taylor series about the response to the mean flow $E(U)$ and ordered in terms of increasing harmonics as

$$E(U+u) = E(U) + \frac{(\omega R)^2}{4} E_{uu} + \frac{(\omega R)^4}{64} E_{uuuu} + \cdots + \cos \omega t \left[(\omega R) E_u + \frac{(\omega R)^3}{8} E_{uuu} + \cdots \right] + \cos 2\omega t \left[\frac{(\omega R)^2}{4} E_{uu} + \frac{(\omega R)^4}{48} E_{uuuu} + \cdots \right] + \cdots \quad (1)$$

The subscript notation indicates partial differentiation $E_u = \partial E/\partial u$. Defining a dimensionless velocity $u^* = u/U$ and performing an order of magnitude analysis, it can be seen that, for $(\omega R/U) \leq 0.5$, Eq. (1) can be written to a good approximation as

$$E(U+u) = E(U) + [(\omega R) E_u] \cos \omega t + \left[\frac{(\omega R)^2}{4} E_{uu} \right] \cos 2\omega t + \cdots \quad (2)$$

It is precisely the relationship in Eq. (2) which is the foundation of the technique. The coefficients of $\cos \omega t$ and $\cos 2\omega t$ can be measured with a lock-in amplifier independent of the level of random backround turbulence. The Taylor series coefficients E_u and E_{uu} can then be determined since ωR, the velocity of the harmonic perturbation, is known. Using these known coefficients, the response of a single wire to an unknown random turbulent velocity perturbation u can be written

$$e(t) = E(U+u) - E(U) = E_u u + (1/2!) E_{uu} u^2 + \cdots \quad (3)$$

where, in Eq. (3), the Taylor series coefficients are known from Eq. (2). The rms of Eq. (3) may be written

$$[\overline{e(t)^2}]^{1/2} = E_u (\overline{u^2})^{1/2} + \frac{E_{uu}}{2(\overline{u^2})^{1/2}} \left\{ \overline{u^3} + \frac{\overline{u^4}}{4} \frac{E_{uu}}{E_u} + \cdots \right\} \quad (4)$$

where the first term is the usual linear response. The magnitude of the nonlinear response may be approximated through the measured coefficients E_u, E_{uu} for a correction to the first-term linear response.

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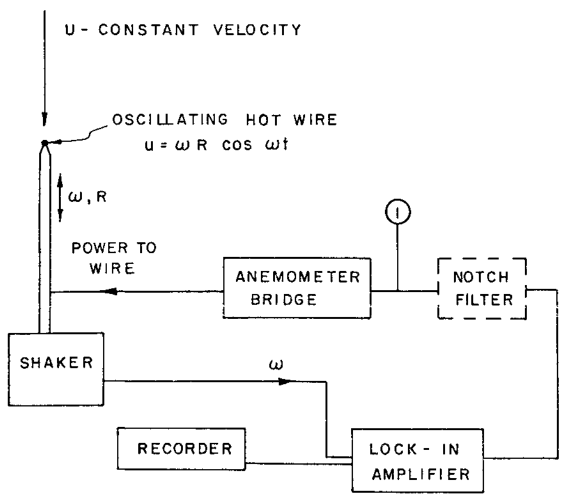

Fig. 1 Experimental schematic.

Experiment and Results

Figure 1 is a line diagram of the experiment. The calibration tests were performed in a constant mean flow velocity field in the potential core of a freejet and in a section of pipe flow. The bride voltage signal at point 1 in Fig. 1 contains harmonics at all frequencies $n\omega$ plus oscillations due to the background level of turbulence in the flow. The notch filter is used to suppress the strong signal at $\cos \omega t$ when the weaker signal at $\cos 2\omega t$ is being detected. The notch filter is not used when the lock-in is tuned to the frequency of forced oscillation ω during the measurement of the coefficient of $\cos \omega t$.

The experiment covered a range of 0.188 in. $\leq R \leq$ 0.406 in. and 10 Hz $\leq f \leq$ 30 Hz for a range of $0.025 \leq \omega R/U \leq 0.45$. Care was taken not to oscillate the wire at a frequency near its natural frequency. The first natural frequency of the wire was approximately 800 Hz as calculated from the determinate for the modes of oscillation of a beam with a step discontinuity.[7]

Figure 2 is a plot of $\partial E/\partial U$ from the static calibration curve $E^2 = A + BU^n$, $n = 0.5$, and $\partial E/\partial u$ from the coefficient of $\cos \omega t$ in Eq. (2) vs the mean flow velocity U. The trend of the data in Fig. 2 is in substantial agreement with the data of Ref. 4 for $\Delta E/\Delta u$. These data in Fig. 2 show no systematic dependence on the parameter $(\omega R/U)$, indicating that E_u is not a function of the frequency of the perturbation. At the higher mean flow speeds

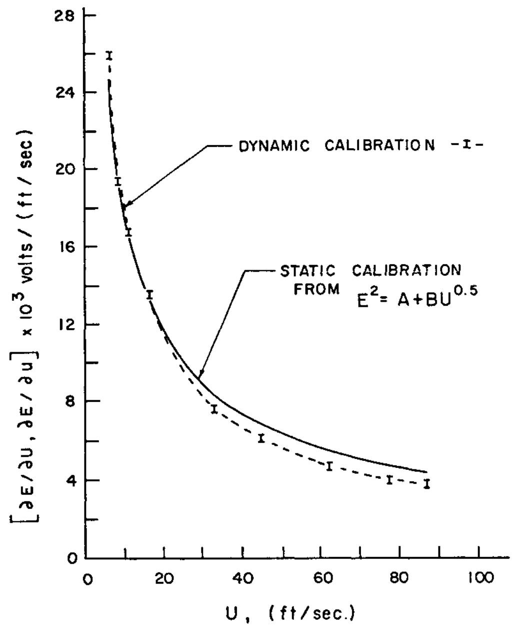

Fig. 2 Linear response terms.

tested, the difference between the static and the dynamic calibration can be as large as 15%. There is no choice of exponent n in King's Law which will produce agreement between the static and dynamic calibration over this range of mean flow velocity.

The measurement of absolute turbulent intensity using the dynamic calibration suggested herein is believed to be more accurate than the static calibration, because the dynamic calibration more correctly models the mode of heat transfer between

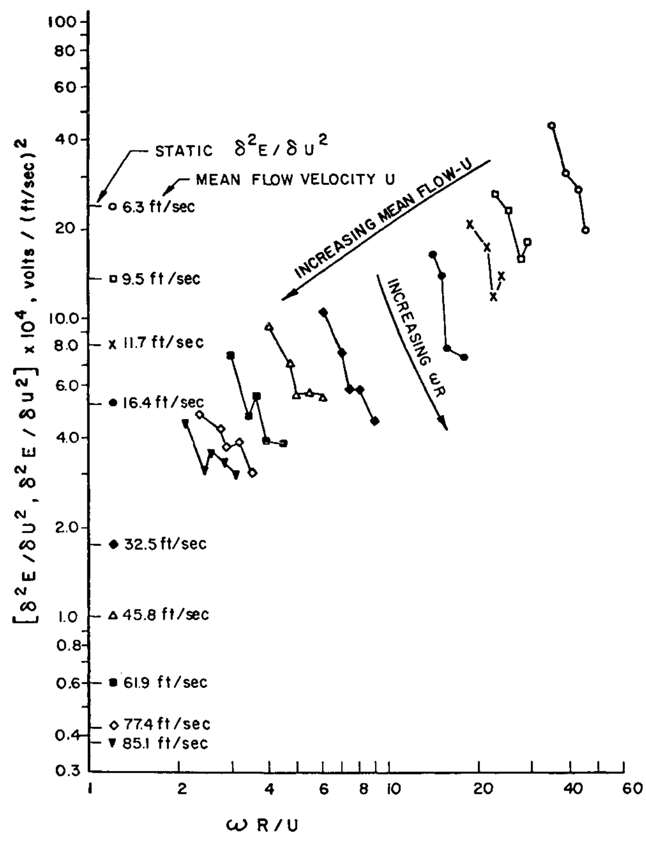

Fig. 3 Second-order response.

the wire and the turbulent flow. There are some data in Ref. 4 to support this conclusion.

Figure 3 is a plot of $\partial^2 E/\partial U^2$ from the static calibration and $\partial^2 E/\partial u^2$ from the coefficient of $\cos 2\omega t$ in Eq. (1) vs the parameter $(\omega R/U)$. The striking feature of these data is the strong dependence of $\partial^2 E/\partial u^2$ on the artificial turbulence level $\omega R/U$. This dependence is not predicted from the second derivative of the static calibration curve $E^2 = A + BU^n$ nor is it observed in the first term of the dynamic response $\partial E/\partial u$. These data indicate that the nonlinear part of the wire response in Eq. (4) is frequency dependent over the range investigated in this experiment. At the higher mean flows, there is considerable discrepancy between the static $\partial^2 E/\partial U^2$ and the dynamic $\partial^2 E/\partial u^2$. Both of these facts contribute to uncertainties in the determination of absolute turbulent intensity.

Conclusions

A technique has been presented which provides for the direct calibration of a hot wire anemometer for turbulence intensity measurements without recourse to the static King's Law response curve. The linear part of the response curve is in agreement with the data of previous workers.[4] The nonlinear part of the response has been found to be dependent on the frequency of the artificial turbulence used to calibrate the probe, indicating that turbulent intensity, as determined from Eq. (4), is a function of its spectral distribution.

Further work is presently underway to determine the bridge voltage response to both longitudinal and transverse velocity perturbations represented by an expansion of $E(U+u, v)$.

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Turbulence Measurements in a Mach 2.9 Boundary Layer Using Laser Velocimetry

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Nomenclature

d_f = distance between interference fringes
d_p = diameter of particle
f_{3db} = frequency related to particle response—3 db point
k = Cunningham constant, 1.8 for air

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